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and Recruiting Decisions in Manpower Planning

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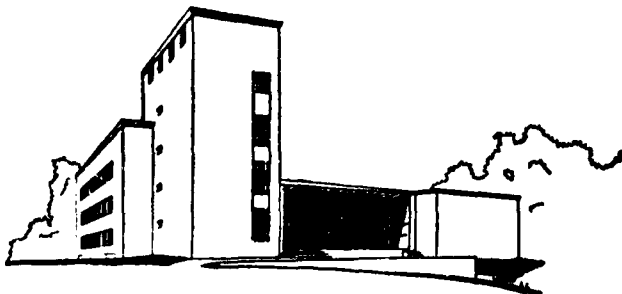
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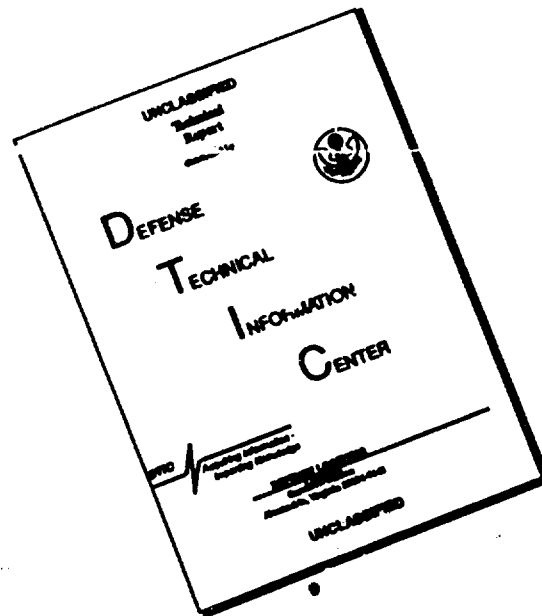
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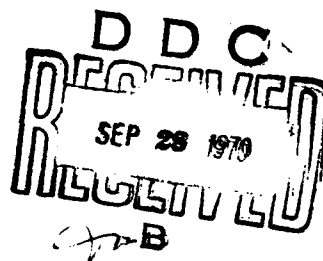
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Abstract

Models for manpower planning previously devised for the U. S. Navy's Office of Civilian Manpower Management have all utilized goal programming constructs with embedded Markoff processes. These models -- referred to as "OCMM Models" -- are here extended to include training elements along with related constraints.

Introduction:

A "goal programming model"^{1/} with iterated Markov elements to allow explicit consideration of transitions, exits, retirements, etc. in manpower planning over a sequence of periods was first introduced in [1]. As noted in the introduction to [1], the initial model formed one part of a total research effort on the part of the U. S. Navy's Office of Civilian Manpower Management (OCMM). This model--which we shall hereafter call the "OCMM Model"--has since been elaborated in a variety of directions including explicit allowance for predicted retirements within certain age or service categories and allowance for dynamically varying Markovian elements from period to period.^{2/} Other parts of the research program enunciated in [1] for the OCMM Models involve relating the multiple goals (e.g., the enunciated manpower ceilings) to tasks that need to be performed as well as introducing training possibilities as alternatives to recruitment and job transfers in order to meet (as closely as possible) the specified goals.^{3/}

In this report we propose to develop a first analytical model which explicitly provides for training as well as outside recruitment and job transition possibilities. We further propose to do this in a way which provides access to a variety of techniques such as

^{1/}See [1] Appendix B and Chapter X for an explanation and development of the ideas of goal programming.

^{2/}See, e.g., [5]

^{3/}Actually some of this has already been done in an earlier phase of this research program--see [8]--and in a way that takes account of on-the-job learning and dynamic organization design (and redesign) to allow for accumulating experience as well as formal training, etc.

"parametric variation" and "duality evaluations"^{1/} in order to facilitate experimentation with manpower program possibilities. In this way we shall be able to bring the power of linear programming to bear in evaluating optimal tradeoff possibilities and their resulting manpower mix and planning consequences. We shall then also be able to coordinate "career management" and "manpower planning" (and other parts of personnel planning)^{2/} by allowing for possible variations in manpower mixes and tradeoff possibilities in recruitment, transfer and training. Naturally, we shall want to accomplish this in a context that also considers other constraints such as financial budgets, supply and recruitment limitations imposed by policy or the environment at various times and also considers, of course, various kinds of limitations on training facilities. All except the last of these constraints have, however, already been included for explicit treatment in one or more versions of the OCMM Models. Hence we, shall here relate these constraints to their predecessor developments in order to be able to utilize some of the results already secured.

2. Modeling Strategies

In a manpower planning context it is natural to want to consider modeling for training in a way that allows for the effects of training and the selection of trainees on the manpower mixes which will be available in the years that follow such training.^{3/} This will be done here in a manner that provides direct contact with the goal programming developments in prior OCMM Models. A convenient way to

^{1/}See [1], Chapters I and VI, for further explanations of these terms^{2/}vide [9]

^{3/}Such effects should, of course, be interpreted as probabilistic projections when Markoff processes are being utilized in planning for a sequence of periods.

accomplish this is to posit that the effect of training may be represented by a different matrix of transition rates which will then apply to personnel who have been selected for training. Then we can split the population into two groups--viz., those selected for training in a specified period and those who are not selected for training. The latter group may then transit in accordance with a "training" matrix while the former transit in accordance with "manpower" matrices of the kind we have previously utilized.^{1/}

With the wanted contacts with previous OCMM Models thus established it is not necessary to treat again all of the previously developed types of constraints and stipulations and their related possibilities of variation and evaluation. Facilities as well as funds limitations on training capacities must be considered, of course, for training possibilities in each period. To continue to relate this to the previous developments, however, we shall here formulate only the funds limitation for explicit consideration. Situations where no facilities are available can then be modeled from a budgetary standpoint as having zero dollars available for that type of training.^{2/} In an analogous manner, period-by-period cost constraints may also be imposed for outside recruitment and, of course, we may then also continue to impose an overall budgetary constraint on salaries, and extend this part of the previous OCMM Models to include costs for recruiting and training as well as salary costs, etc., to be considered in each period, as desired

^{1/}See [3] and [5] where similar devices were used to obtain refinements for retirement and related considerations. The latter developments, including extensions to dynamically varying Markoff elements are, of course, also available for use in these OCMM Models (as noted in our introduction).

^{2/}Alternatively, a lack of facilities can also be reflected in the training transition matrix by introducing zero rates of transition into certain parts of matrix and various combinations of budget and transition rate possibility may also be employed, of course

In this same spirit we shall also preserve contact with the previous goal programming developments in our formulation of this model's objective. Thus we shall specify one part of this objective in terms of meeting a stipulated collection of manpower planning goals "as closely as possible", while staying within the constraints specified for each period in the planning horizon. We shall also extend the previously utilized objectives by including additional elements directed toward minimizing the total costs of outside recruitment and internal training. This extension is of interest in its own right, of course, but it also has the additional advantage of providing further insight for evaluating the relative weights assigned to deviations from other goals along with simultaneous evaluations of training-recruitment costs and tradeoffs.^{1/}

Finally we shall also want to utilize the types of transformations and reductions available from our prior research in order to develop special structures which would otherwise remain latent and, perhaps, unutilized for the computational advantages that such special structures can supply.^{2/} Indeed, the formulations we shall employ will give rise to a model structure which further generalizes the "network type" model relations that have been elaborated in [1]^{3/}. In fact, we may regard the developments which we shall employ here as representing still further generalizations of the generalized network models presented in [1] but, to avoid a proliferation of terminology, we shall refer to this model, too, as a "generalized network type model."

^{1/}Of course, total salary and related cost considerations will also be available for evaluation, too, via the budgetary constraints.

^{2/}See, e.g., the discussion of "model types" in [1] Chapter 1 and ff.

^{3/}Vide, e.g., Chapter XVII in [1].

3. Definitions and Development of Generalized Network Type

The above modeling strategy may be given analytical form as follows. Let^{1/}

$$(1.1) \quad x_{ij}(t) = \text{number of personnel assigned to "job type i" from "source j," without assignment to additional training in period t}$$

$$y_{ij}(t) = \text{number of personnel assigned to "job type i" from "source j" with assignment to training in period t}$$

so that

$$(1.2) \quad x_{ij}(t) + y_{ij}(t) = \text{total number of personnel to be assigned to job type i from source j in period t.}$$

Thus, as indicated in the preceding section, the number of personnel obtained from source j may be summed and further distinguished between those assigned directly to job type i and those assigned to training for job type i. This will be done for each of the periods $t = 0, 1, 2, \dots, N$ comprehended in the horizon for which manpower planning is being undertaken.

At time $t = 0$, the number of persons already in job type i may be represented by a known constant

$$(2) \quad a_i = \text{number of personnel in job type i within the organization at } t = 0.$$

^{1/} As explained in [2], the terms "job type" and "source" are intended to comprehend distinctions between claimants or activities (for the same job or position) and recruitment or assignment in different geographical regions--and possibly other characteristics, too, if desired.

Furthermore, to simplify notation we may consider a_i as one of the components of a vector "a" comprehending all pertinent job types and, correspondingly, let

$$(3) \quad x^j(t) = \begin{bmatrix} x_{1j}(t) \\ \vdots \\ x_{nj}(t) \end{bmatrix}, \quad y^j(t) = \begin{bmatrix} y_{1j}(t) \\ \vdots \\ y_{nj}(t) \end{bmatrix}$$

represent vectors with n components for each of the job types $i = 1, \dots, n$.

In proceeding toward our model objectives we shall want to allow for transfers between job types and also for the possibility that persons recruited for training in one category may subsequently transit to some other category. Thus, in keeping with previous developments, we introduce the Markoff matrix^{1/}, M, with elements

$$(4) \quad M_{ij} = \begin{matrix} \text{proportion of those in job } j, \text{ without training in} \\ \text{job } j, \text{ who will transit to job } i \end{matrix}$$

Then we introduce another matrix, T, with elements

$$(5) \quad T_{ij} = \begin{matrix} \text{proportion of those in job } j \text{ with training in job } j \\ \text{who will transit to job } i \end{matrix}$$

In order to bring the desired type of generalized network relations into prominence, we proceed as follows. For any period t,

^{1/}Actually we will modify the usual Markoff representation and, just as we have done before, we omit one row and column to allow for the fact that in these OCM Models entrance into the system is to be determined by reference to decisions on recruitment. Note, unlike other manpower planning and analyses which use Markoff processes, the decision variables and objectives are set forth explicitly along with other constraints, including policy limitations, etc.

we can let

$x(t)$ = vector of personnel within the organization
who are not being trained.^{1/}

(6.1) $y(t)$ = vector of personnel within the organization
who are being trained.

$z(t)$ = vector of personnel from outside the organization
who are being brought in.

Then we introduce the following types of relations,

$$(6.2) \quad z(1) + Mx(0) + Ty(0) = x(1) + y(1)$$

wherein $z(1)$ represents the vector of personnel recruited from outside while $Mx(0) + Ty(0)$ represents the transfer via jobs and training from inside and the whole splits into the two new vectors $x(1)$ for personnel not being trained and $y(1)$ for personnel who are being trained in period 1. The sum $x(1) + y(1)$ then represents the total number employed in each job type in period 1.

An evident extension of these developments now produces

$$\begin{aligned} a &= x(0) + y(0) \\ 0 &= -Mx(0) - Ty(0) + x(1) + y(1) && -z(1) \\ (7.1) \quad 0 &= && -Mx(1) - Ty(1) + x(2) + y(2) && -z(2) \end{aligned}$$

and so on via

$$\begin{aligned} 0 &= -Mx(t-1) - Ty(t-1) + x(t) + y(t) && -z(t) \\ 0 &= && -Mx(t) - Ty(t) + x(t+1) + y(t+1) && -z(t+1) \end{aligned}$$

^{1/}See [8] for developments in which job type assignments are also effected for their training potential on other types of jobs at subsequent times.

where $z(0) \equiv 0$.^{1/} Reference to this structure suggests incidence relations of the generalized network variety. Of course, the indicated incidences are on vectors $x(t)$, $y(t)$ and matrices M and T but the representations (7.1) and (7.2) nevertheless display a structure which lends itself to this symbolism and related interpretations as a further generalization of these generalized network concepts and developments which we have previously used to advantage for serving computational efficiencies. See [1] Chapter XVII ff.

4. Additional Constraints and Objectives:

Other constraints will also be needed, however, to allow for limitations on training and recruitment. As indicated in preceding sections, we want to relate these to previously utilized formulations of budgetary limitations in OCMM Models. Thus, we continue from (6.1) and introduce the following "training constraints"

$$(8) \quad K^1(t) y(t) \leq d^1(t)$$

where $K^1(t)$ is a matrix of the costs for training for each job type and source at time t and $d^1(t)$ is a stipulated vector of limitations imposed on such expenditures.

In this same vein we may also represent the constraints on outside recruitment via

$$(9) \quad K^2(t) z(t) \leq d^2(t)$$

where $K^2(t)$ is a matrix of recruitment costs at time t and $d^2(t)$ is a corresponding vector of stipulations.

^{1/}Evidently we can also replace M and T by time-dependent Markoff matrices as in [5] and [8].

Finally we insert budget constraint on total salaries, as in previous OCMM Models,

$$c^T(t) [x(t) + y(t)] \leq B(t)$$

where

$$(10) \quad c^T(t) = \text{transpose of the vector of salaries to be paid for each job type in period } t$$

$$B(t) = \text{budget limitation (a scalar) on total salaries which may be paid in period } t.$$

The above constraints may be adjoined to those exhibited in the preceding section. Then letting

$$f_k(t) = \text{prescribed ceiling for } k^{\text{th}} \text{ type of manpower.}$$

$$(11) \quad \mu_{kt} = \text{weight assigned to deviation from } k^{\text{th}} \text{ manpower ceiling in period } t$$

we can formulate the objective for this model as

$$(12) \quad \min \sum_{t=1}^N \sum_k \mu_{kt} \left| e_{I_k}^T [x(t) + y(t)] - f_k(t) \right| + \sum_{t=1}^N \mu_{ot} [\alpha^T y(t) + \beta^T z(t)]$$

where α^T and β^T represent vectors (transposed) containing the recruiting and training costs elements to be considered in the objective while

$$(13) \quad e_{I_k}^i \equiv \begin{pmatrix} 0 & , & 1 \\ ixI_k & , & ixI_k \end{pmatrix}$$

is a vector which has zeros in all components except those which are the unity elements. The latter, i.e., the values of unity, are in

the positions that correspond to the k^{th} type of manpower.

5. Matrix and Structure:

Drawing the elements of the preceding two sections together we may obtain the matrix represented in Table 1. This may be used for developing direct or dual relations if desired but here we have only utilized the direct variable $x(t)$, $y(t)$ and $z(t)$ as defined and developed in the preceding section.^{1/} Such extensions will require replacing the (nonlinear) absolute value terms in (12) by their linear programming equivalent via the usual "goal programming" reductions.^{2/} This has been done in the preceding papers in this series, however, and hence need not be repeated here.

With this structure now being available the stage is set for further interpretation and extensions. This will be done in a supplemental report, however, and made more concrete by means of a simple numerical example and just as was done for the reports finally incorporated in [5] this will be accompanied by related computations and solution results. The portrayal in Figure 1, for the time being, then completes the present report.

^{1/}These developments implicitly utilize transformations and reductions first introduced in [5] for simplifying matters and making the underlying structure apparent as a guide to computational and interpretative developments.

^{2/}See Chapter X in [1] for a general development including geometric interpretations and analytical developments of the theory underlying these reductions.

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